

Fuzzy Set Theory, Intersection, and its Applications

Sajid Ahmed¹, Ansari Zaid Anjum², Nadeem Anwer³

^{1, 2}(Department of APSC, MMANTC/ University of Pune, India)

³(Department of Mathematics, Dr. D. Y. Patil School of Engineering/ University of Pune, India)

Abstract: The objectives of this fuzzy sets theory is that of evaluating some of the difficulties coming in the study of Western logic paradigm obtaining by Aristotelian logic. This logic suggest a binary truth value which guides to the idea known as law of excluded middle which states that stated anyhypothesis either it or its negation must be true. In terms of set theory this convert into a binary introduction of elements in a set so that given set A, either $a \in A$ or $a \notin A$ and $a \in \bar{A}$. Such 'a' idea, however, begets to many popular contradictions normally grouped under the class of the rule of contradiction which propose that if we believe the hypothesis that separate a seed from a pile of sand does not source it to inactive its state of being a pile, the consecutive transfer of one seed at a time construct us categories one seed as a pile of sand if we permit the shifting of a particular seed to trigger the change in categorization of the mass to "some seeds of sand", which is not realistic. Other main feature which fuzzy set theory and fuzzy logic try to mark is how to deal with the intrinsic vagueness of regularly used words such as excellent, high, good which cannot be defined in traditional (or crisp) set theory without introducing un reasonable bounce in closeness of the border of the classes.

Keywords: classes, fuzzy intersection, fuzzy set etc.

I. Introduction

The first publications in fuzzy set theory by Zadeh [1965] and Goguen [1967, 1969] show the purpose of the authors to discover the classical notion of a set and a proposition to accommodate fuzziness in the sense.

L. A. Zadeh [1965] writes, "The imagination of a fuzzy set gives a convenient point of going for the construction of a hypothetical which parallels in multiple respects the framework utilized in the case of ordinary sets, but is more general than latter & potentially, may show to have a much bigger scope of applicability, especially in the area of pattern classification and dataprocessing. Basically, such a framework gives a natural way of handling with (FUZZY SET THEORY-AND ITS APPLICATIONS) problems in which the origin of imprecision is the absence of clearly defined criteria of class membership rather than presence of random variables." "Imprecision" here is meant in the sense of indeterminate rather than the short of knowledge about the value of a parameter. Fuzzy set theory gives a tight mathematical framework (there is nothing fuzzy about fuzzy set theory) in which indeterminate conceptual phenomena can be exactly and tightly studied. It can also be useful as a modeling language well-formed for situations in which fuzzy relations, criteria, and phenomena exist.

II. Definition of Fuzzy Sets

A fuzzy subset of a universal set X is a mathematical object 'A' stated by its characteristic function (membership function) $\mu_A : X \rightarrow [0, 1]$. The classical membership degrees are denoted by 1 (is a member) and 0 (not a member)

Alternative notation: A(x)

F(X) denotes the set of all fuzzy subsets of a universal set X.

III. Fuzzy Intersection

The idea of intersection in the fuzzy set framework depends upon that of triangular norm (commonly read as t-norm). A t-norm is a generalization of intersection for lattices and can be explained in other functional forms as long as the following fundamental properties are guaranteed.

- 1) Commutativity: $t(a;b) = t(b;a)$
- 2) Monotonicity: $t(a;b) \leq t(c;d)$ if $a \leq c$ and $b \leq d$
- 3) Associativity: $t(a; t(b;c)) = t(t(a;b);c)$
- 4) Neutrality of 1: $t(a; 1) = a$

Additionally in fuzzy logic the t-norm is also need to be a continuous function. Given these properties, different types of t-norms have been suggest to enforce the intersection in fuzzy set theory and 3 of them are

by far the most powerful. The definitions are here stated in words of membership functions of $x \in X$ w.r.t. fuzzy sets a , b and $c = a \cap b$:

1) Gödel –Dummett t-norm

$$f_c(x) = \min(f_a(x), f_b(x))$$

2) Product t-norm

$$f_c(x) = (f_a(x) \cdot f_b(x))$$

3) Lukasiewicz t-norm

$$f_c(x) = \max(0, f_a(x) + f_b(x) - 1)$$

The results comes operating different t-norms on the same application case can be look in some of the examples provided in a review paper by Isabelle Bloch.

IV. Applications

As we know before many of the application found in research are concentrated on fuzzy logic. Fuzzy set theory is used in some interesting fields to enhance the outcome obtained with crisp sets. What makes these applications more interesting is the reality that they include far extra attention to the theoretical basis of fuzzy sets and have good algebraic and mathematical base than those centered on fuzzy logic. The applications presented are mathematical morphology, photo analysis (regarding spatial relational connectivity between objects) and photo processing. The researcher is encouraged to read the papers in order to achieve a detailed insight into the real implementations.

V. Final Conclusion

In this work the important vision of the fuzzy intersection have been reported on. While it is by no means exhaustive from the point of view of the desirable applications, this report was meant to give the theoretical basis which is compulsory to proceed to more advanced topics, with the help of some examples on how these logic have been used in research so far.

References

- [1]. <http://en.wikipedia.org/wiki/t-norm>. Accessed on 30/07/2010.
- [2]. I. Bloch and H. Maître. Fuzzy mathematical morphologies: a comparative study. *Pattern Recognition*, 28(9):1341-1387, 1995.
- [3]. I. Bloch. Fuzzy spatial relationships for image processing and interpretation: a review. *Image and Vision Computing*, 23:89-110, 2005.
- [4]. K. Chakrabarty, R. Biswas and S. Nanda. A note on fuzzy union and fuzzy intersection. *Fuzzy Sets and Systems*, 105:499-502, 1999.
- [5]. E.E. Kerre and M. Nachtgael. Fuzzy relational calculus and its applications to image processing. In *Fuzzy Logic and Applications, volume 571 of LNCS, pages 179-188: Springer Berlin / Heidelberg*, 2009.
- [6]. L. A. Zadeh. Fuzzy sets. *Information and Control*, 8(3):338-353, 1965.